Sequential Edge Linking on SAR images: Two Novel Metrics

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➢ General Context

➢ Need for edge linking phase after edge detection stage

➢ Original SEL algorithm problems

➢ Proposed solutions

➢ Experimental results

➢ Conclusions
Context: Landmark Extraction

- **Civilian Applications:**
  - Water course monitoring;
  - Damage evaluation caused by natural calamities.

- **IMINT Applications:**
  - Monitoring and Change Detection;
  - Route analysis.

- **Aeronavigation Applications:**
  - Position and attitude of an aircraft can be retrieved by exploiting the linear landmark recognition in the SAR image acquired by the airborne radar.

**Landmark:** prominent object that univocally marks the land acquired by SAR.
Edge Detection Problems

- Segmentation of SAR images: Edge Detection is usually the first step.
- Due to the speckle noise, classical edge detectors (gradient operators) are not CFAR:
  - Greater probability of false alarms (PFA) on image areas with higher values.

MSTAR CLUTTER Dataset

<table>
<thead>
<tr>
<th>Radar Type</th>
<th>Airborne</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>X-band</td>
</tr>
<tr>
<td>Range Res.</td>
<td>0.3m</td>
</tr>
<tr>
<td>Cross-Range Res.</td>
<td>0.3m</td>
</tr>
<tr>
<td>Polarization</td>
<td>HH</td>
</tr>
<tr>
<td>Incidence Angle</td>
<td>75°</td>
</tr>
<tr>
<td>Acquisition Mode</td>
<td>StripMap</td>
</tr>
<tr>
<td>ENL</td>
<td>1</td>
</tr>
</tbody>
</table>

- Even using CFAR edge detectors: many interruptions and missed edges.

**Solution:**
- Edge Linking stage.
The linking issue is modeled as a Shortest Path Problem (SPP):
- Which is the best path? A, B, or C?
- Do these paths exist?

Solving the shortest path for each end point is time expensive.

Solution:
- SEL algorithm:
  - Only three neighbor pixels of the actual best path are explored.
  - The cost to reach them is put on a priority queue, and the best path is always at the top.
  - The final path has a high probability to be near the optimum path.
- A path is defined as: 
  \[ p = \{z_0, d_0, \{a_i\}_{i=1}^{n}\} \]

- Path shape is controlled by defining conditional probabilities:
  \[ P(p) = P(a_i, a_{i-1}, \ldots, a_1) = P(a_i|a_{i-1})P(a_{i-1}|a_{i-2}) \cdots P(a_2|a_1)P(a_1) \]

- Edge following is controlled by Likelihood Ratio:
  \[ Q(p) = \frac{P_{H_1}(z_1, \ldots, z_n)}{P_{H_0}(z_1, \ldots, z_n)} = \frac{\prod_{i=1}^{n} P_{H_1}(z_i)}{\prod_{i=1}^{n} P_{H_0}(z_i)} \]

- Final Metric:
  \[ \gamma(p) = P(p)Q(p) = \prod_{i=1}^{n} P(a_i|a_{i-1}) \frac{\prod_{i=1}^{n} P_{H_1}(z_i)}{\prod_{i=1}^{n} P_{H_0}(z_i)} \]
**Proposed Parametric Metric**

**Final SEL Metric:**

\[ \gamma(p) = P(p)Q(p) = \prod_{i=1}^{n} \frac{P_H(z_i)}{P_H(z_i)} \]

**Problem:**

- After the edge detection stage both \( P_{H_0}(z_i) \) and \( P_{H_1}(z_i) \) are hardly known.

**Proposed Solution:**

- Using the RoA edge detector.

**OBS:** \( \bar{I}_i \) is the Maximum Likelihood (ML) estimate of \( \mu_i \)

\[ r_{i,\theta} = \frac{\bar{I}_1}{\bar{I}_2} \]

Presupposing a Gamma PDF of the image \( I \):

\[ P_{H_0}(z_i) = \frac{\Gamma(2NL)}{\Gamma(NL)^2} \left( r_{i,\theta} \right)^{NL-1} \frac{\mu_1}{\mu_2+N} \left( \frac{\mu_1}{\mu_2+r_{i,\theta}} \right)^{2NL} \]

\[ P_{H_1}(z_i) = \frac{\Gamma(2NL)}{\Gamma(NL)^2} \left( \frac{\mu_1}{\mu_2} \right)^{NL} \left( \frac{\mu_1}{\mu_2+r_{i,\theta}} \right)^{NL-1} \]

\( L \) : Number of looks

\( N \) : Number of pixels in each part of the window

\( \mu_i \) : RCS inside the part \( i \) of the window
Proposed Parametric Metric

- **Final SEL Metric:**
  \[
  \gamma(p) = P(p)Q(p) = \prod_{i=1}^{n} P(a_i | a_{i-1}) \cdot \frac{\prod_{i=1}^{n} P_{H_1}(z_i)}{\prod_{i=1}^{n} P_{H_0}(z_i)}
  \]

- **Problem:**
  - After the edge detection stage both \( P_{H_0}(z_i) \) and \( P_{H_1}(z_i) \) are hardly known.

- **Proposed Solution:**
  - Using the RoA edge detector.

**Information Merging:**
\[
R_{i,\theta} = \max\left(r_{i,\theta}, r_{i,\theta}^{-1}\right) \quad \theta^* = \arg\max_{\theta} R_{i,\theta}
\]

Window with the maximum probability of having detected an edge.
Proposed Parametric Metric

**Final SEL Metric:**
\[ \gamma(p) = P(p)Q(p) = \prod_{i=1}^{n} P(a_i|a_{i-1}) \prod_{i=1}^{n} P_{H_1}(z_i) \prod_{i=1}^{n} P_{H_0}(z_i) \]

**Problems:**
- After few steps the probability to be in a state \( a_n \) is not longer tied to the initial transition probabilities;
- Many path loops appear applying this metric on SAR images.

**Proposed Solution:**
- Linking the path direction to the gradient direction estimated by the edge detector.

\[ P(p) = P(\theta_i, ..., \theta_0 | \hat{\theta}_i, ..., \hat{\theta}_0) \propto P(\theta_i | \hat{\theta}_i)P(\theta_{i-1} | \hat{\theta}_{i-1}) \cdots P(\theta_0 | \hat{\theta}_0) \]

\( \hat{\theta}_i \): estimated gradient direction at pixel \( Z_i \)
\( \theta_i \): direction of the path at pixel \( Z_i \)

*Independent \( \hat{\theta}_i \);
Independent \( \theta_i \);
First order Markov process.*

**Example**
- \( P(\theta_i = \hat{\theta}_i | \hat{\theta}_i) = 0 \)  
- \( P(\theta_i = \hat{\theta}_i \pm \frac{\pi}{4} | \hat{\theta}_i) = 1/3 \)  
- \( P(\theta_i = \hat{\theta}_i \pm \frac{\pi}{2} | \hat{\theta}_i) = 2/3 \)

\( \theta_i, \hat{\theta}_i \in [0, \frac{3\pi}{4}] \)
Parameters:
- RoA
  \( L = 1 \)
  11x11 window
  4 directions
- Non Maxima Supp.
- Thresholding
  \( PFA_{high} = 10^{-13} \)
  \( PFA_{low} = 10^{-9} \)
The previous *parametric* metric can be applied only when data follow a Gamma PDF.

**Proposed Solution:**
- *Non-parametric* generalization which can be applied to any (differentiable) image.
- *Proposed Metric* comes from «snakes» theory:

\[ E = E_{\text{int}} + E_{\text{ext}} \]

- \( E_{\text{int}} \) controls the path shape and can map some a priori information.

\[ E_{\text{int}} = \int_{\mathbb{R}} [\alpha |c'| + \beta |c''|] dt \]

\( \alpha \) : elasticity of the curve
\( \beta \) : rigidity of the curve

- \( E_{\text{ext}} \) keeps the path near the boundaries:

\[ E_{\text{ext}} = \int_{C} f_1 (\nabla I \cdot \mathbf{k}) dC + \int_{C} f_2 (\nabla I \cdot \mathbf{n}) dC \]

\[ + \int_{C} f_1 (|\nabla I| \cdot \mathbf{\hat{v}}_1 - |\nabla I| \cdot \mathbf{\hat{v}}_2) dC + \int_{C} f_2 (|\nabla I|) dC \]

\( f_1(x) = \frac{1}{1 + |x|} \)

\( f_2(x) = \frac{|x|}{1 + |x|} \)

\( \mathbf{\hat{k}} \) : unit vector tangential to the curve \( C \)
\( \mathbf{\hat{n}} \) : unit vector orthogonal to the curve \( C \)
\( \mathbf{\hat{v}}_1, \mathbf{\hat{v}}_2 \) : unit vector at \( \pm \frac{\pi}{4} \) from \( \mathbf{\hat{k}} \)
The previous parametric metric can be applied only when data follow a Gamma PDF.

**Proposed Solution:**
- Non-parametric generalization which can be applied to any (differentiable) image.
- Proposed Metric comes from «snakes» theory:
  \[ E = E_{\text{int}} + E_{\text{ext}} \]

- \( E_{\text{int}} \) controls the path shape and can map some a priori information.
  \[ E_{\text{int}} = \int_{\mathbb{R}} [\alpha |c'| + \beta |c''|] dt \]
  \( \alpha \) : elasticity of the curve
  \( \beta \) : rigidity of the curve
  \( c(t) \): parameterization of \( C \)
  \( c(t) = (x(t), y(t)) \)

- \( E_{\text{ext}} \) keeps the path near the boundaries:
  \[ E_{\text{ext}} = \int_{C} f_1(\nabla I \cdot \hat{k}) dC + \int_{C} f_2(\nabla I \cdot \hat{n}) dC \]
  + \[ \int_{C} f_1(|\nabla I \cdot \hat{v}_1| - |\nabla I \cdot \hat{v}_2|) dC \]
  + \[ \int_{C} f_2(|\nabla I|) dC \]

\( \hat{k} \) : unit vector tangential to the curve \( C \)
\( \hat{n} \) : unit vector orthogonal to the curve \( C \)
\( \hat{v}_1, \hat{v}_2 \) : unit vector at \( \pm \frac{\pi}{4} \) from \( \hat{k} \)

\( \nabla I \) should be orthogonal to \( \hat{k} \) and parallel to \( \hat{n} \)

Directional derivatives along \( \hat{v}_1 \) and \( \hat{v}_2 \) should be equal (in module)
\[ |\nabla I| \] should be greater than possible
**Comparison:**

**Performance Evaluation**

### Circle vs. Square

#### Simulated
- Gamma i.i.d.
- \( L = 1 \)
- RCS ratio = 3

#### Non Parametric
- \( \alpha = \beta = 0 \)

#### Parametric
- \( P(\theta_i = \bar{\theta}_i | \bar{\theta}_i) = 0 \)
- \( P(\theta_i = \bar{\theta}_i \pm 1/3 | \bar{\theta}_i) = 1/3 \)
- \( P(\theta_i = \bar{\theta}_i \pm 2/3 | \bar{\theta}_i) = 2/3 \)

<table>
<thead>
<tr>
<th></th>
<th>Circle</th>
<th>Square</th>
<th>Circle</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MD</strong></td>
<td>0.76</td>
<td>0.69</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td><strong>FOM</strong></td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**OBS:** Derivative is applied after a logarithmic transformation.

### Observation
- **SEL** algorithm can be used for a ROI extraction: starting from the strongest edge value, it reconstructs a closed boundary.

- **Parametric** metric outperforms **Non Parametric** one on curvilinear boundaries (Circle) but they are nearly equal on straight edges (Square).

### Both Metrics
- Both metrics are always near to the optimum path.

**OBS:** The optimum path has been computed fixing the metric and looking for the path which is the best among all possible paths.
Comparison: Performance Evaluation

Parametric metric obtains the best results:
- Curvilinear boundaries;
- Long interruptions.

Non Parametric metric manages to reconstruct the main edges.

Computational Time

- **CPU 2.53GHz Intel Core2 Duo, RAM 1,99GB.**
- **Image 1024x1024 px.**
- **Environment MATLAB**

<table>
<thead>
<tr>
<th>Metric</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>RoA 11x11</td>
<td>~100s</td>
</tr>
<tr>
<td>Derivative 11x11</td>
<td>~1s</td>
</tr>
<tr>
<td>SEL</td>
<td>~10s</td>
</tr>
</tbody>
</table>

SEL: ~10ms for each endpoint
Conclusions

- On SAR images, the speckle noise drastically reduces the performance of edge detector algorithms.

- Solution: SEL algorithm after the edge detection stage.

- Original SEL metric shows many application problems (likelihood ratio computation, path loops, useless transition probabilities).

- We propose two possible solutions to SEL:
  - Parametric metric relying on the RoA edge detector;
  - Non-parametric metric relying on a general derivative operator.

- Both of them use information derived from the image gradient.

- Parametric metric yields the best performance even though the RoA edge detector is quite computational expensive.

- Non-parametric metric obtains good results and it is much faster than parametric one.
THANKS FOR YOUR ATTENTION...

...ANY QUESTION?